Performing a Safe Propulsive Landing at Model Scale Using Solid Propellant Engines

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Abstract

To perform a safe landing, a model rocket must use one of many methods of slowing its descent. Common methods of achieving such deceleration are parachutes, streamers, or gliding (winged) recoveries. However, some space agencies in the real world use a method of slowing descent called propulsive landing. Propulsive landing is the process of employing thrust forces to slow the rocket before landing. The maneuver involves the activation of a downward facing engine before landing in order to bring the vehicle to a near complete halt at the time of touchdown.

There are many different reasons for the usage of propulsive landing technology.. Experimenting with model rockets provides a great substitute at a much cheaper price than using a full scale rocket to experiment with key components of propulsive landing.

The intent for writing this document is to explore the benefits, drawbacks, and applications of using propulsive landing in both the model rocketry and real world.

SYMBOLS

μ_D	$\frac{1}{2}\rho C_D A$
A	Rocket cross sectional area
ρ	Air density
C_D	Rocket's coefficient of drag, assume 0.75 (Benson, n.d)
F	Force
t	Time, duration, or burn time of engine
I	Impulse ($F * t$)
m	Mass of rocket
ME	Mechanical energy
W	Work

Additional symbols, when used, are defined in the text.

SUBSCRIPTS

$()_i$	Initial value (ex. $v_f = v_i + at$)
$()_f$	Final value (ex. $v_f = v_i + at$)
$()_{\#}$	Value of ( ) during stage # (ex. $v_{i_1} = 0$)
$t_{\#}$	Duration of stage (ex. $t_1 = \frac{I_1}{F_1}$)

Additional subscripts, when used, are defined in the text.

SECTION 1: Introduction and Background

A model rocket is a replica of a real rocket, sharing many key features. A model rocket is subject to forces like aerodynamic drag, gravity, thrust. A major difference between model and real rockets is the ease of calculating rotational forces acting on a larger rocket, since it is much easier for more advanced data collection and predictions to be done on a larger scale rocket's less-jolty motion.

But why use propulsive landing instead of a more fuel efficient method, such as a parachute-assisted descent? In most cases, the answer is that other methods aren't actually more fuel efficient, since a parachute to support the weight of a large rocket would need to be immensely massive. This can be modeled by the following example scenario:

> A safe landing speed for a rocket is roughly 20 km/h, or 5.5 m/s. (O'Connell, 2018) Therefore, for a rocket to maintain a velocity of at most 5.5 meters per second, the drag force exerted by the pressure must be greater than or equal the weight of the rocket:
>
> $$F_g \leq F_{Drag}$$
>
> $$mg \leq \frac{1}{2}\rho v^2 C_D A$$
>
> $$9.8m \leq \frac{1}{2}\rho(-5.5)^2 C_D A$$
>
> The value for the drag coefficient of a parachute is about 1.75 (Benson, n.d.), and the air density at sea level is about $1.225\ kg/m^3$ (*Machine Applications Corporation*, 2019). The mass of a Falcon 9, which is to be used as an example, is 549,054 kg (*Falcon 9*, n.d.), resulting in the following formula:
>
> $$5380729.2 \leq 32.4A$$
>
> $$\text{Cross sectional area of parachute} = 166071.9\ m^2$$

Therefore, the cross sectional area of the parachute must be at least 166071.9 square meters in order to maintain a safe landing speed of at most 5.5 meters per second. Assuming that the parachute density is 0.035 kg per square meter (*Parachute Fabric*, n.d.), the total weight of the parachute comes out to 5808 kilograms, or 5.8 metric tons, which is considerably heavy.

Let us take a step back to the question of why propulsive landing might be used instead of a more traditional descent method. Real space agencies answer this question using some of the following criteria (EagerSpace, 2022):

- Is it worth the cargo capacity / range compromise?

Since cargo space is limited, the additional mass of the extra propellant used to perform a propulsive landing could significantly reduce maximum cargo capacity, or maximum range. Therefore, it is important to evaluate whether it is worth the compromise of the two mentioned fields. See Table 1 below.

Orbit	Max Cargo Mass Without Using Propulsive Landing	Max Cargo Mass Using Propulsive Landing	% loss of maximum cargo capacity to orbit
Low Earth	22,800 kg	16,250 kg	28.7%
Geostationary Transfer	8,300 kg	5,500 kg	33.7%

Table 1: Maximum cargo mass to be injected into different orbits, with or without using propulsive landing technologies, for the Falcon 9 rocket (EagerSpace, 2022)

- Is the minimum thrust to weight ratio (TWR) of the rocket low enough to allow for a propulsive landing maneuver?

  According to a document published by SpaceX in 2021, the throttle of a single Falcon 9 Merlin engine ranges from 481.7 kN to 845.2 kN (SpaceX, 2021). Therefore, the lowest thrust to weight ratio (TWR) possible is 481.7 to 549, or 0.88. On the other hand, its maximum TWR (maximum throttle on all engines) is 7686 to 549, or 14. Do note that these two values assume that the rocket is at full propellant capacity. Therefore, since the minimum throttle provides a low enough thrust, which enables more precision than a higher minimum TWR.

- Is it cost efficient?

  The money and time required to research, design and implement a propulsive landing system could outweigh the benefits of using such a system. Some rockets, operated by certain contractors, are used very few times to be profitable from using a reusable rocket.

SECTION 2: Assumptions

The flight, assuming it is perfectly vertical with no horizontal component to its motion, consists of 4 main stages:

Stage 1 - Powered Ascent) From t = 0 to engine 1 burnout.

- The rocket is accelerating on the +Y axis, and its velocity is also directed upwards. Drag (F_D) acts on the -Y axis.
- Mass lost from burning propellant is not significant enough to take into account in a real-world scenario.

Stage 2 - Unpowered Ascent) From engine 1 burnout to the maximum height of the motion (the vertex, see fig. 1 below)

- The rocket is accelerating on the -Y axis, and its velocity is directed upwards for the entirety of this stage
- Drag continues to act on the -Y axis, opposite to the direction of its velocity.

Stage 3 - Ballistic descent) From the vertex to the engine 2 burn initiation time;

- The rocket continues to accelerate on the -Y axis. However, its velocity is directed downwards, as it continues to free fall.
- Drag now acts on the +Y axis, assisting in slowing the rocket down

Stage 4 - Powered descent) Engine 2 burn initiation to touchdown.

- Acceleration, with the thrust force from engine 2, is directed on the +Y axis, and the velocity approaches 0 as the rocket approaches the ground.

The amount of stages that the motion of the rocket is divided into must be at least 4, with those being as described above. However, having more stages corresponds to having more precision throughout the calculations (Smith, 2024)

Figure 1: SpaceX Falcon 9 rockets using propulsion to slow their descent (From SpaceX on Flickr)

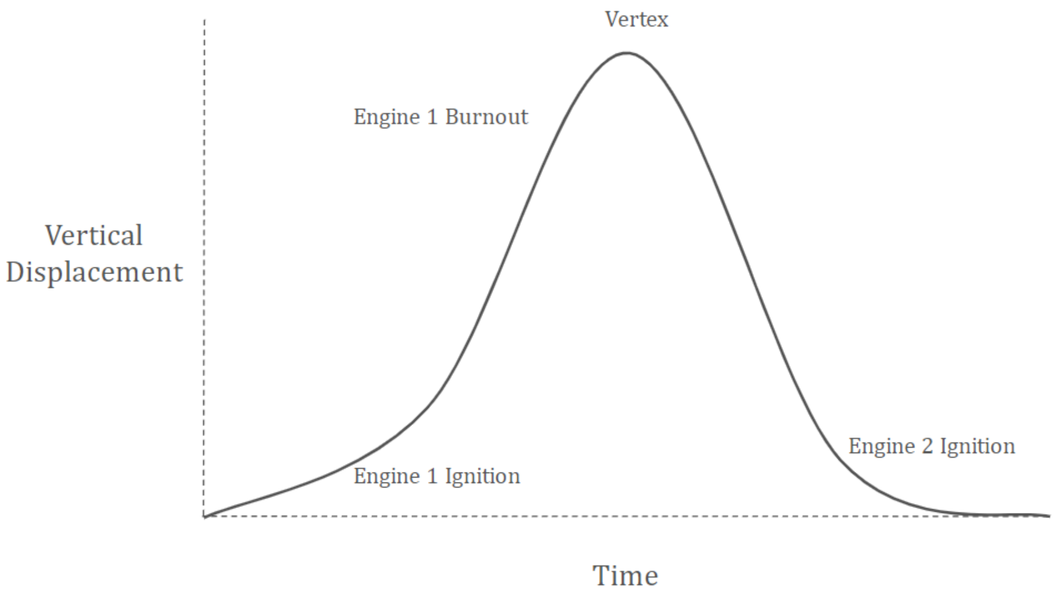

Figure 2-1: Approximation of vertical displacement as a function of time.

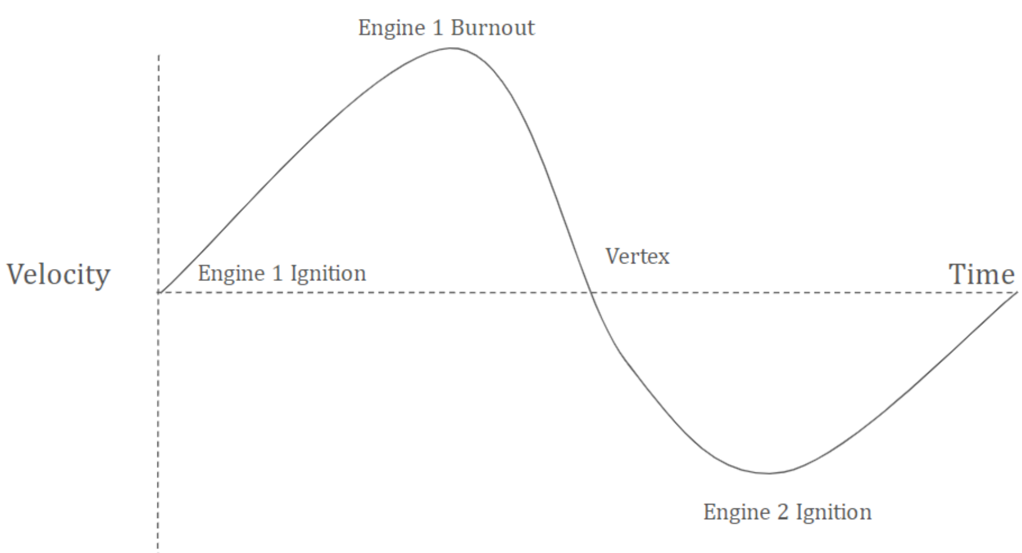

Figure 2-2: Approximation of velocity as a function of time

Do note that the above figures aren't symmetrical about $x = time / 2$. Mechanical energy is conserved, but air resistance is a non-conserved quantity. Therefore, the minimum velocity in the velocity graph is not equal to the maximum velocity. Due to this, as will be discussed later in this document the impulse of the solid-propellant descent engine must be less than the one used for liftoff, so as to avoid the vehicle beginning to fly again after coming to a stop upon landing.

A mathematical model to represent the above scenario is as follows:

$$\Delta ME \ = \ 0$$
$$ME_i \ = \ ME_f$$
$$W_{Engine_1} + W_{Drag} = W_{Engine_2}$$

Since work done by drag is negative, the following must be true:

$$W_{Engine_1} - \left| W_{Drag} \right| = W_{Engine_2}$$
$$W_{Engine_1} > W_{Engine_2}$$
$$Impulse_{Engine_1} > Impulse_{Engine_2}$$

SECTION 3: Derivation of the time of ignition

The initial step to derivating t_{ignite} is to solve for the speed of the rocket after completing the burn of engine 1 and entering stage 1:

$$v_f = \ v_i + \ at$$

$$F_{net} \ = \ ma \therefore a \ = \ F_{net} \,/\, m$$

$$F_D = \frac{1}{2} C_D \rho v^2 A$$

Since the force of air resistance is exponentially proportional to velocity, an expression for velocity at every instant during the course of the motion is required. There are multiple ways to achieve this, such as writing an expression for the velocity during any given part of the motion with respect to dt. A good approximation of velocity at any instant would be assuming it is equal to the average velocity of a stage:

$$v_{avg} = \frac{v_f + v_i}{2}$$

Therefore, solving for v_f:

$$v_f = 0 + \left(\frac{F_1}{M} - \frac{\mu_D\left(\frac{v_i + v_f}{2}\right)^2}{m} - g\right)t_1,$$

which can be simplified as

$$\frac{4mv_f}{t_1} = 4F_1 - \mu_D v_f^{\ 2} - 4mg.$$

The equation must be set to equal 0 in order to solve for v_f with the quadratic formula:

$$(\mu_D)v_f^{\ 2} + \left(\frac{4m}{t_1}\right)v_f + (4F_1 - 4mg) = 0.$$

The values for the coefficients are:

$$a = \mu_D,\ b = \frac{4m}{t_1},\ c = 4F_1 - 4mg$$

therefore,

$$v_f = \frac{-\frac{4m}{t_1} \pm \sqrt{\frac{16m}{t_1^{\ 2}} + (4)(\mu_D)(4F_1 - 4mg)}}{2\mu_D}.$$

In this formula, gravity acts on the -Y axis, therefore, v_f must be positive at the end of the stage. So, we will take only the positive zero of v_f, and simplifying:

$$v_1 = \frac{-\frac{4m}{t_1} + \sqrt{\frac{16m}{t_1^{\ 2}} + 16\mu_D F_1 - 16\mu_D mg}}{2\mu_D}.$$

Then, in order to derive for t_{ignite}, it is necessary to know the time it takes for gravity and drag to decelerate the rocket from $v_{f_{stage1}}$ to a halt. The reason for having to solve for stage 2 and stage 3 separately is because air resistance acts on the -Y axis before reaching maximum height, and on the +Y axis as the rocket falls towards the ground. The point where the direction of the work done by air resistance switches is right after the point where velocity reaches 0 after liftoff, known as the vertex of the motion. Now knowing that the final velocity for stage 2 is 0, and that its initial velocity is the final velocity of stage 1, it is possible to set up an equation for the time taken to complete stage 2.

Using the kinematic formula $v_f = v_i + at$, t_2 can be solved for:

$$0 = v_1 + (-g + \mu_D(\frac{0+v_1}{2})^2)t_2$$

$$0 = v_1 - (g + \mu_D(\frac{v_1^2}{4}))t_2$$

$$t_2 = \frac{v_1}{g+\mu_D(\frac{v_1^2}{4})}$$

Solving for t of stage 3 requires to solve for the v_i of stage 4. Therefore, once again, using kinematic $v_f = v_i + at$, we solve for v_i of stage 4, which needs to come to a stop at the end, before hitting the ground:

$$0 = v_i + (-g + \mu_D(\frac{0+v_i}{2})^2)\, t_4$$

$$(\frac{1}{4}\mu_D t_4)v_i^2 + v_i - gt_4 = 0$$

$$a = \frac{1}{4}\mu_D t_4,\ b = 1,\ c = -gt_4$$

$$v_i = \frac{-1 \pm \sqrt{1-(4)(\frac{1}{4}\mu_D t_4)(-gt_4)}}{2\frac{1}{4}\mu_D t_4}$$

$$v_i = \frac{-2 \pm 2\sqrt{1-(\mu_D t_4)(-gt_4)}}{\mu_D t_4}$$

Taking the negative root, as velocity will be negative at descent,

$$v_{i_4} = \frac{-2 - 2\sqrt{1-(\mu_D t_4)(-gt_4)}}{\mu_D t_4}.$$

Finally, solving for t of stage 3 using the same kinematic formula:

$$v_{i_4} = v_2 + \mu_D(\frac{v_2+v_{i_4}}{2})^2 t.$$

$$\frac{v_{i_4}}{\mu_D\left(\frac{v_{i_4}}{2}\right)^2} = t.$$

So, the time of igniting the second motor in order to achieve a slow speed upon landing would be

$$t_1 + t_2 + t_3$$

Plugging in calculated values for each, the final ignition time comes out to

$$t_{ignite} = t_{engine_1} + \frac{v_1}{g+\mu_D\left(\frac{v_1^2}{4}\right)} + \frac{v_{i_4}}{v_2+\mu_D\left(\frac{v_{i_4}}{2}\right)^2},$$

where $v_{i_4} = \frac{-2-2\sqrt{1-(\mu_D t_4)(-gt_4)}}{\mu_D t_4}$ and $v_1 = \frac{-\frac{4m}{t_1}+\sqrt{\frac{16m}{t_1^2}+16\mu_D F_1-16\mu_D mg}}{2\mu_D}$

SECTION 4: References and Citations

Benson, T. (n.d.). *Terminal Velocity*. Terminal Velocity. Retrieved January 21, 2024, from https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/termvr.html

Benson, T. (n.d.). *Velocity During Recovery*. Velocity During Recovery. Retrieved March 4, 2024, from https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/rktvrecv.html

Delcotex. (n.d.). *Parachute fabric | PIA.C-44378 Type III*. Delcotex. Retrieved March 4, 2024, from https://delcotex.de/en/products/parachute-fabric-piac-44378-type-iii

EagerSpace. (2022, May 5). *Propulsive Landing - What does it take?* YouTube. Retrieved March 3, 2024, from https://www.youtube.com/watch?v=8eDS2UhT88I&ab_channel=EagerSpace

MAC Instruments. (2019, November 21). *What Is the Density of Air at STP? - Machine Applications Corporation*. MAC Instruments. Retrieved March 4, 2024, from https://macinstruments.com/blog/what-is-the-density-of-air-at-stp/

O'Connell, C. (2018, February 6). *Reusable rockets explained*. Cosmos Magazine. Retrieved March 4, 2024, from https://cosmosmagazine.com/space/launch-land-repeat-reusable-rockets-explained/

Smith, Jeff. (2024, January 8). Interview. Conducted by Bora Ayhan.

SpaceX. (n.d.). *Falcon 9*. SpaceX. Retrieved March 4, 2024, from https://www.spacex.com/vehicles/falcon-9/

SpaceX. (2018, February 7). *Falcon Heavy Demo Mission | Official SpaceX Photos*. Flickr. Retrieved March 5, 2024, from https://www.flickr.com/photos/spacex/25254688767/

SpaceX. (2021, September). *Falcon User's Guide*. Falcon User's Guide. Retrieved March 8, 2024, from https://www.spacex.com/media/falcon-users-guide-2021-09.pdf